

Mathematica Lab 3

Uppgift 1

$$\tilde{A} = \frac{1}{2} \begin{pmatrix} -10 & -6 & 12 \\ 8 & -4 & -18 \\ k & 2 & 4 \end{pmatrix};$$

$$\tilde{A} = \frac{1}{3} \begin{pmatrix} -18 & -12 & -39 \\ 9 & 6 & 21 \\ k & -10 & -7 \end{pmatrix};$$

inv1 = Inverse@AD

$$98-6, -4, -13<, 83, 2, 7<, 9-\frac{10}{3}, -\frac{7}{3}, -\frac{22}{3}==$$

"Bryter ut" 1/3:

inv1 * 3 •• MatrixForm

$$\begin{pmatrix} -18 & -12 & -39 \\ 9 & 6 & 21 \\ k & -10 & -7 \end{pmatrix}$$

Uppgift 2

Clear@"`*"D

$$A = \begin{pmatrix} k & k & -2 \\ 1 & k & 1 \\ k & 1 & k \end{pmatrix};$$

Inverse@AD •• MatrixForm

$$\begin{pmatrix} \frac{k^2-1}{k^3+2k^2-k-2} & \frac{-k^2-2}{k^3+2k^2-k-2} & \frac{3k}{k^3+2k^2-k-2} \\ 0 & \frac{k^2+2k}{k^3+2k^2-k-2} & \frac{-k-2}{k^3+2k^2-k-2} \\ \frac{1-k^2}{k^3+2k^2-k-2} & \frac{k^2-k}{k^3+2k^2-k-2} & \frac{k^2-k}{k^3+2k^2-k-2} \end{pmatrix}$$

Det@AD

$$k^3 + 2k^2 - k - 2$$

Matrisen är endast inverterbar om Det[A] ≠ 0;

ekv2 = Det@AD == 0;

Solve@ekv2D

$$88k \rightarrow -2<, 8k \rightarrow -1<, 8k \rightarrow 1<<$$

Om k=-2 eller -1 eller 1 är matrisen inte inverterbar

Uppgift 3

```

Clear@"`*"D
ekv3a = 3 x - 2 y + z + 2 w == 12;
ekv3b = x - 3 y + 2 z + w == 14;
ekv3c = 2 x + y - z + w == -2;
ekv3d = x + 4 y - 3 z == -16;
Solve@8ekv3a, ekv3b, ekv3c, ekv3d<, 8x, y, z, w<D

Solve::svars : Equations may not give solutions for all "solve" variables.

99x  $\rightarrow -\frac{1}{7} H4 wL + \frac{z}{7} + \frac{8}{7}, y \rightarrow \frac{w}{7} - \frac{30}{7} + \frac{5 z}{7} ==$ 

koeffmatris =  $\begin{pmatrix} 3 & -2 & 1 & 2 \\ 1 & -3 & 2 & 1 \\ 2 & 1 & -1 & 1 \\ 1 & 4 & -3 & 0 \end{pmatrix} y$ ;

Det@koeffmatrisD

0

```

Ajdå, Det[]=0 det betyder ej entydig lösning!

```

totalmatris =  $\begin{pmatrix} 3 & -2 & 1 & 2 & 12 \\ 1 & -3 & 2 & 1 & 14 \\ 2 & 1 & -1 & 1 & -2 \\ 1 & 4 & -3 & 0 & -16 \end{pmatrix} y$  •• RowReduce •• MatrixForm

 $\begin{pmatrix} 1 & 0 & -\frac{1}{7} & \frac{4}{7} & \frac{8}{7} \\ 0 & 1 & -\frac{5}{7} & -\frac{1}{7} & -\frac{30}{7} \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} y$ 

```

Jaha, de två sista säger att 0=0 och det visste vi väl redan!

Således, 2 ekv med 4 obekanta => två parametrar:

```

Solve@8ekv3a, ekv3b, ekv3c, ekv3d, y == 5 t, w == 5 s<, 8x, y, z, w<D •• Simplify

88x  $\rightarrow 2 - 3 s + t, y \rightarrow 5 t, z \rightarrow 6 - s + 7 t, w \rightarrow 5 s <<$ 

```

Uppgift 4

```

Clear@"`*"D
ekv4a = p * x + y + 2 z + w == p;
ekv4b = x + p * y + 2 z + w == p;
ekv4c = x + 2 y + p * z + w == p;
ekv4d = x + y + 2 z + p * w == p;
Solve@8ekv4a, ekv4b, ekv4c, ekv4d<, 8x, y, z, w<D

99x  $\rightarrow \frac{p}{4 + p}, y \rightarrow \frac{p}{4 + p}, z \rightarrow \frac{p}{4 + p}, w \rightarrow -\frac{p - p^2}{H - 1 + pL H4 + pL} ==$ 

```

$$A = \begin{pmatrix} p & 1 & 2 & 1 \\ 1 & p & 2 & 1 \\ 1 & 2 & p & 1 \\ 1 & 1 & 2 & p \end{pmatrix};$$

Det@A

$$p^4 + 18p - 11p^2 - 8$$

$$\text{ekv4e} = p^4 - 11p^2 + 18p - 8 == 0;$$

Solve@ekv4eD

$$88p \rightarrow -4 <, 8p \rightarrow 1 <, 8p \rightarrow 1 <, 8p \rightarrow 2 <<$$

Svar: För entydig lösning p -4,1,2. det ger:

Solve@8ekv4a, ekv4b, ekv4c, ekv4d, p != -4, p != 1, p != 2 <, 8x, y, z, w <D •• Simplify

$$99x \rightarrow \frac{p}{4+p}, y \rightarrow \frac{p}{4+p}, z \rightarrow \frac{p}{4+p}, w \rightarrow \frac{p}{4+p} ==$$

Nu testar vi vad x,y,z,w blir om vi stoppar in p som 1,2,-4:

Solve@8ekv4a, ekv4b, ekv4c, ekv4d, p == 1, z == t, w == s <, 8x, y, z, w <D •• Simplify

$$88x \rightarrow 1 - s - 3t, y \rightarrow t, w \rightarrow s, z \rightarrow t <<$$

Solve@8ekv4a, ekv4b, ekv4c, ekv4d, p == 2, w == s <, 8x, y, z, w <D •• Simplify

$$88x \rightarrow s, y \rightarrow s, z \rightarrow 1 - 2s, w \rightarrow s <<$$

Solve@8ekv4a, ekv4b, ekv4c, ekv4d, p == -4 <, 8x, y, z, w <D •• Simplify

$$8 <$$

Saknas lösning!

Uppgift 5

Clear@"`*"D

$$A = \begin{pmatrix} 3 & 2 & 4 \\ 1 & 6 & 5 \\ 3 & 9 & 1 \end{pmatrix}; \quad \tilde{A} = \begin{pmatrix} 16 & 21 \\ 25 & 27 \\ 34 & 16 \end{pmatrix};$$

$$x = \begin{pmatrix} a & d \\ b & e \\ c & f \end{pmatrix}$$

$$\text{ekv5} = A \cdot x == \tilde{A};$$

Solve@ekv5D

$$88a \rightarrow 2, b \rightarrow 3, c \rightarrow 1, d \rightarrow 1, e \rightarrow 1, f \rightarrow 4 <<$$

Observera matrismultiplikationen!

Svar: $x = \begin{pmatrix} 2 & 1 \\ 3 & 1 \\ 1 & 4 \end{pmatrix}$

Uppgift 6

```
Clear@"`*"D
A = J  $\begin{pmatrix} 3 & 1 \\ 5 & -2 \end{pmatrix}$  N; A = J  $\begin{pmatrix} 14 & 13 \\ 33 & 57 \end{pmatrix}$  N; x = J  $\begin{pmatrix} a & c \\ b & d \end{pmatrix}$  N;
ekv6 = 2 A . x == A + 3 x;
Solve@ekv6D

88a → 4, b → 1, c → 5, d → -1<<
```

Svar: $x = \begin{pmatrix} 4 & 5 \\ 1 & 1 \end{pmatrix}$

Uppgift 7

```
Clear@"`*"D
A =  $\begin{pmatrix} 3 & 5 & -1 \\ 1 & 7 & 5 \\ -5 & -13 & 6 \\ 1 & -17 & 9 \end{pmatrix}$ ; x =  $\begin{pmatrix} x \\ y \\ z \end{pmatrix}$ ; A =  $\begin{pmatrix} -60 \\ 19 \\ 219 \\ 322 \end{pmatrix}$ ;
ekv7 = A . x == A;
```

Enligt boken:

```
A t = Transpose@A D;
normalekv = A t . A . x == A t . A;
Solve@normalekv, 8x, y, z<D •• N

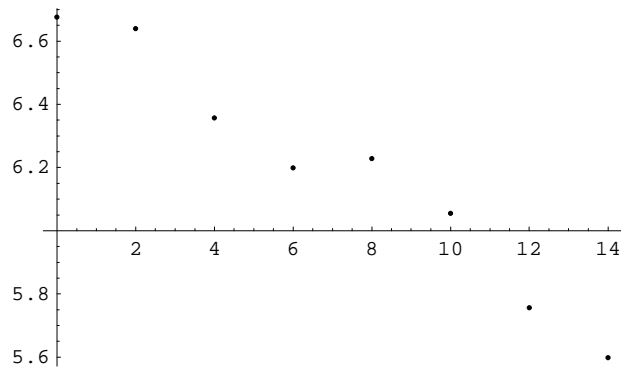
88x → 2.08021, y → -9.77983, z → 17.0684<<
```

Uppgift 8

```

Clear@"`*"D
data = 881982, Log@793D<, 81984, Log@764D<, 81986, Log@579D<, 81988, Log@492D<,
      81990, Log@507D<, 81992, Log@426D<, 81994, Log@316D<, 81996, Log@270D<<;
år = 80, 2, 4, 6, 8, 10, 12, 14<;
ant = 8793, 764, 576, 492, 507, 426, 316, 270<;
data2 = Table@8år@@yDD, Log@ant@@yDDD<, 8y, 1, 8<D;
ListPlot@data2D

```



- Graphics -

```
y = f@t_D = Fit@data2, 81, k * t<, tD
```

```
6.72288 - 0.0763745 t
```

```
x = Ey
```

```
E6.72288-0.0763745 t
```

```
Print@"Svar: aL I=", x ** Simplify, " där t = 0 är 1982"D
```

```
Svar: aL I=831.21 E-0.0763745 t där t = 0 är 1982
```

```
b@t_D = x;
```

```
b@18D;
```

```
Print@"Svar: bL ", b@18DD
```

```
Svar: bL 210.217
```

```
b@t_D == 100 ** Solve
```

```
Solve::ifun : Inverse functions are being used by Solve, so some solutions may not be found.
```

```
88t_ -> 27.728<<
```

```
Print@"Svar cL ", 1982 + 28D
```

```
Svar cL 2010
```