Svar till valda uppgifter i Johnson & Wichern 1998

11.1 (a) Den linjära diskriminantfunktionen i (11.19) är

$$\hat{y} = (\bar{\boldsymbol{x}}_1 - \bar{\boldsymbol{x}}_2)' S_{\text{poolad}}^{-1} \boldsymbol{x} = \hat{\boldsymbol{a}}' \boldsymbol{x}$$

där

$$S_{\text{poolad}}^{-1} = \begin{pmatrix} 2 & -1 \\ -1 & 1 \end{pmatrix}$$

Den linjära diskriminantfunktionen blir då

$$(\begin{pmatrix} 3 \\ 6 \end{pmatrix} - \begin{pmatrix} 5 \\ 8 \end{pmatrix})' \begin{pmatrix} 2 & -1 \\ -1 & 1 \end{pmatrix} \boldsymbol{x} = (-2, 0)\boldsymbol{x} = -2x_1$$

(b)

$$\hat{m} = \frac{1}{2}(\hat{y}_1 + \hat{y}_2) = \frac{1}{2}(\hat{\boldsymbol{a}}'\bar{\boldsymbol{x}}_1 + \hat{\boldsymbol{a}}'\bar{\boldsymbol{x}}_2) = -8$$

Tilldela $\boldsymbol{x}_0$ till π_1 om

$$\hat{y}_0 = (-2, 0)\boldsymbol{x}_0 \geq \hat{m}_0 = -8$$

och tilldela $\boldsymbol{x}_0$ till π_2 annars.

Eftersom $(-2, 0)\boldsymbol{x}_0 = -4 > \hat{m}_0 = -8$ tilldelas $\boldsymbol{x}_0$ till π_1.

11.2

11.3

11.4

11.5

11.7

11.8

11.9

$$\frac{\boldsymbol{\ell}' B_0 \boldsymbol{\ell}}{\boldsymbol{\ell} \Sigma \boldsymbol{\ell}} = \frac{\boldsymbol{\ell}'[(\boldsymbol{\mu}_1 - \bar{\boldsymbol{\mu}})(\boldsymbol{\mu}_1 - \bar{\boldsymbol{\mu}})' + (\boldsymbol{\mu}_2 - \bar{\boldsymbol{\mu}})(\boldsymbol{\mu}_2 - \bar{\boldsymbol{\mu}})']\boldsymbol{\ell}}{\boldsymbol{\ell} \Sigma \boldsymbol{\ell}}$$

där $\bar{\boldsymbol{\mu}} = (\boldsymbol{\mu}_1 + \boldsymbol{\mu}_2)/2$. Alltså blir

$$\frac{\boldsymbol{\ell}' B_0 \boldsymbol{\ell}}{\boldsymbol{\ell} \Sigma \boldsymbol{\ell}} = \frac{\boldsymbol{\ell}'[(\boldsymbol{\mu}_1 - \boldsymbol{\mu}_2)(\boldsymbol{\mu}_1 - \boldsymbol{\mu}_2)']\boldsymbol{\ell}}{\boldsymbol{\ell} \Sigma \boldsymbol{\ell}}$$

11.10

11.11

11.16

11.19