Svar till valda uppgifter i Johnson & Wichern 1998

10.1
$$\Sigma_{11}^{-1/2}\Sigma_{12}\sigma_{22}^{-1}\Sigma_{21}\Sigma_{11}^{-1/2} = \begin{pmatrix} 0 & 0 \\ 0 & 0.95^2 \end{pmatrix}$$

har egenvärdena $\rho_1^2 = 0.95^2$ och $\rho_2^2 = 0$. De normaliserade egenvektorerna är

$$\boldsymbol{e}_1 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \quad \boldsymbol{e}_2 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

Då blir

$$U_1 = \boldsymbol{e}_1'\Sigma_{11}^{1/2}\boldsymbol{X}^{(1)} = (0,1)\begin{pmatrix} 0.1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} X_1^{(1)} \\ X_2^{(1)} \end{pmatrix} = X_2^{(1)}$$

Eftersom $\boldsymbol{f}_1'\Sigma_{22}^{-1/2} = (1,0)$ blir $V_1 = X_1^{(2)}$.

Alltså: $U_1 = X_2^{(1)}, V_1 = X_1^{(2)}$ och $\rho_1 = 0.95$.

10.2 (i) $\rho_1 = 0.55, \rho_2 = 0.49$.

(ii) $U_1 = 0.32X_1^{(1)} - 0.36X_2^{(1)}$, $V_1 = 0.36X_1^{(2)} - 0.10X_2^{(2)}$.

10.3

10.4

10.5

10.6

10.7 (i) $\rho_1^* = \frac{2\rho}{1+\rho}$, $0 < \rho < 1$.

$$U_1 = \frac{1}{\sqrt{2(1+\rho)}}(X_1^{(1)} + X_2^{(1)})$$

$$V_1 = \frac{1}{\sqrt{2(1+\rho)}}(X_1^{(2)} + X_2^{(2)})$$

10.9

10.10